a) $f(x)=e^{x/2}$

b) $\frac{e^x-1}{x}$

$e^x = 1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\dots+\frac{x^n}{n!}\dots$

$\frac{e^x-1}{x} = x+\frac{x^2}{2x}+\frac{x^3}{3!x}+\dots+\frac{x^n}{n!x}\dots$

$= 1+\frac{x}{2}+\frac{x^2}{3!}+\dots+\frac{x^{n-1}}{n!}$

c) $g' = \frac{xe^x-(e^x-1)}{x^2} = \frac{1}{2}+\frac{2x}{3!}+\dots+\frac{(n-1)x^{n-2}}{n!}$

$g'(1) = \frac{e-e+1}{1} = 1 = \frac{1}{2}+\frac{2}{3!}+\dots+\frac{n-1}{n!}$

$1 = \frac{1}{2}+\frac{2}{3!}+\dots\frac{n}{(n+1)!}$

Feb 3-8:42 AM

9.2b Taylor Series

Maclaurin series you should have memorized:

$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots+\frac{x^n}{n!}$

$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}\dots\frac{(-1)^n x^{2n+1}}{(2n+1)!}$

n starts at 0

$\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}\dots\frac{(-1)^n x^{2n}}{(2n)!}$

$\ln(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}\dots\frac{(-1)^{n+1}x^n}{n}$

n starts at 1

Jan 31-5:13 PM

Taylor series for f(x) centered on x=a

centered on $x=0$ — Maclaurin Series

$f(x) = f(0)+f'(0)x+\frac{f''(0)x^2}{2}\dots\frac{f^{n}(0)x^n}{n!}$

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centered on $x=a$ — Taylor series

$$\boxed{f(x) = f(a)+f'(a)(x-a)+\frac{f''(a)(x-a)^2}{2!}\dots\frac{f^{n}(a)(x-a)^n}{n!}}$$

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Jan 31-5:16 PM

Find the Taylor series generated by f(x)=e^x at x=2

$f(x)=e^x \quad f(2)=e^2$

$f'(x)=e^x \quad f'(2)=e^2$

$f''(x)=e^x \quad f''(2)=e^2$

$f'''(x)=e^x \quad f'''(2)=e^2$

$f^4(x)=e^x \quad f^4(2)=e^2$

$e^2+e^2(x-2)+\frac{e^2(x-2)^2}{2!}\dots\frac{e^2(x-2)^n}{n!}$

Jan 31-5:17 PM

Find the Maclaurin series for $f(x)=\frac{1+\cos(2x)}{2}$

$\cos x = 1-\frac{x^2}{2}+\frac{x^4}{4!}\dots\frac{(-1)^n x^{2n}}{(2n)!}\dots$

$\frac{1+\cos(2x)}{2} = \frac{1+1}{2}-\frac{(2x)^2}{2\cdot 2}+\frac{(2x)^4}{2\cdot 4!}\dots\frac{(-1)^n(2x)^{2n}}{2(2n)!}$

Jan 31-5:18 PM

Find the first four nonzero terms for the Maclaurin series for $\sin(x^2)$

Use this series to find a corresponding series for $\int\sin(x^2)\,dx$

(why not just evaluate $\int\sin(x^2)\,dx$?)

$\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}$

$\sin(x^2) = x^2-\frac{(x^2)^3}{3!}+\frac{(x^2)^5}{5!}-\frac{(x^2)^7}{7!}$

$\int\sin(x^2) \approx \int x^2-\frac{x^6}{3!}+\frac{x^{10}}{5!}-\frac{x^{14}}{7!}$

$\int\sin(x^2)dx = \frac{x^3}{3}-\frac{x^7}{7\cdot 3!}+\frac{x^{11}}{11\cdot 5!}-\frac{x^{15}}{15\cdot 7!}\dots$

Jan 31-5:20 PM